Quiz on Derivatives

Friday 14th, October

Name:

Net ID:

Section:

1. State the definition of the derivative of a function $f(x)$. (2pts)

The derivative is a function

$$f'(x) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

It's the slope of point x at the intersection with the graph of $f(x)$

2. Compute the following derivatives using any tricks you know. (2pts each)

(a) $f(x) = 3x^2 + 5x - 12$

$6x + 5$

Using the power rule $(x^n)' = x^n \cdot \ln n$, we get $(3x^2 + 5x - 12)' =$

$= 3 \cdot 2x + 5 \cdot 1 = 6x + 5$

(b) $f(x) = 2e^4$

Since $(e^x)' = e^x$,

$f'(x) = 2e^4$

~~Using the power rule,~~ $\cancel{(2e^4)' = 8e^3}$

Answer: 0.

(c) $f(x) = \sin(x) \cdot \ln(x)$

Product rule: $(fg)' = f'g + g'f$

$-\cos(x) \cdot \ln x + \sin(x) \cdot \ln x =$

$= (\sin x - \cos x) \cdot \ln x$

Using chain rule,

$\cos x \cdot \ln x - \frac{\sin x}{x}$

(d) $f(x) = \ln(\cos(x))$

$\frac{1}{\cos x}$

$\cancel{(\ln \cos x)'} = \cancel{\frac{1}{\cos x}} \cdot \cancel{(-\sin x)}$?

$\boxed{\cancel{f'(x) = \frac{1}{\cos x}}}$?

(e) $f(x) = \frac{e^x}{x}$

$\left(\frac{e^x}{x}\right)' = \frac{\cancel{x} \cdot e^{x-1}}{\cancel{x}} = e^{x-1}$

Using the quotient rule,

$\left(\frac{e^x}{x}\right)' = e^x \cdot \left(\frac{1}{x}\right)' + \left(e^x\right)' \cdot \frac{1}{x}$

$= e^x\left(-\frac{1}{x^2} + \frac{1}{x}\right) = \frac{e^x(x-1)}{x^2}$

3. Let $f(x)$ be a function, a some real number, and h a variable. Decide what best represents the following expressions (delete as appropriate), and use a few of your own words to describe the expression given. (2pts each)

$f(x)$	Function/Number	function
$f(a)$	Function/Number	number
The slope of $f(x)$ at a	Function/Number	slope ?
The tangent line to $f(x)$ at a	Function/Number	line at a point f(x)
$f'(x)$	Function/Number	derivative
$f'(a)$	Function/Number	derivative ?
$\frac{f(x+h)-f(x)}{h}$	Function/Number	derivative ?

4. True or false: $\frac{d}{dx}(\ln(\pi)) = \frac{1}{\pi}$. (2pts)

Since $(\ln x)' = \frac{1}{x}$, get

$\ln(\pi)' = \frac{1}{\pi}$. True

Using chain rule,

$\ln(\pi)' = \frac{1}{\pi} \cdot \pi' = 0$

Answer : 0

5. Using the chain rule, show that the derivative of $\ln(x)$ is $1/x$. That is, start with the composition

$$e^{\ln(x)=x}$$

and differentiate it using the chain rule. Do not use implicit differentiation, and do not use the formula for the derivative of the inverse function. (6pts)

$e^{\ln x} = x$ Derivative:

$\left(e^{\ln x}\right)' = x' = 1$.

$\left(e^{\ln x}\right)' = \ln x \cdot e^{\ln x - 1} \cdot (\ln x)' =$

$= \cancel{\ln x} \cdot \frac{e^{\ln x}}{\cancel{\ln x}} \cdot (\ln x)'$

$= e^{\ln x} \cdot (\ln x)'$

$= x \cdot (\ln x)' = 1$

So $\boxed{(\ln x)' = \frac{1}{x}}$

$e^{\ln x} = x$

$\left(e^{\ln x}\right)' = x'$

$(\ln x)' = \frac{1}{x}$

By chain rule,

$\left(e^{\ln x}\right)' = \ln x \cdot \left(e^{\ln x}\right)' = \ln x$?

get $\left(e^{\ln x}\right)' = (\ln x)' \cdot \left(e^{\ln x - 1}\right) \cdot \ln x$

$\cancel{(\ln x)' = \left(e^{\ln x}\right)'} =$

$\left(e^{\ln x}\right)' = (\ln x)' \cdot e^{\ln x} = (\ln x)' \cdot x = 1$

$(\ln x)' \cdot x = 1$

so $(\ln x)' = \frac{1}{x}$